Quantifying the priority placed on scale-free smartphone actions

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Abstract

A hallmark of human behavior is the ability to choose from a set of available actions. According to one theory humans execute the task with the highest priority at each decision point Barabasi (2005); Vázquez et al. (2006). The easy access to smartphones forces us to decide between using the phone or performing some other activity that does not involve the phone. Here we show that the priority placed on the phone shapes the temporal dynamics of the behaviour across multiple time scales and we estimate the perceived importance of touchscreen actions in 84 individuals. Heavy-tailed power-law distribution of inter-event times emerged from repeating a simple decision process that decided between smartphone actions and all other actions. The shape of this distribution was determined by the allocated priority of smartphone actions such that the higher the priority on the smartphone over any other actions the fewer the longer gaps across multiple time scales. Quantification of real touchscreen activities showed a heavy-tailed power law distribution of inter-event times ranging from a second to several hours. Over the sampled population, the mean power-law exponent is 1.82 ($\pm$0.12) and we estimate that 4.8% of the population consider touchscreen tasks to be more important than any other activity.

Results

Intervals between smartphone touches are scale invariant

The first aim of our study is to objectively quantify the priority that each individual places on their smartphone. The second aim is to find a non-arbitrary priority threshold such that in a given population, we can state what is the fraction of individuals who place a higher priority on their smartphone than any other activity. Here, we recorded the timing of the touchscreen touches from 84 individuals over a month long period. What is the priority theses individuals place on their smartphones?

A naive possibility would be to simply count the number of touches per day N and claim that higher the N more the priority individuals place on their smartphone. However, the relationship between N and the priority placed on the smartphone is not obvious and any threshold N^* would arbitrarily demarcate individuals who place a higher priority on the smartphone than any other activity. So this proposition would fall short of the 2 aims listed above.

Here, we take the perspective that the intervals between touches carry the relevant information to characterise the level of priority placed on the smartphone. In particular, the presence of a lot of long inter-touch intervals (ITI) would be a sign that the smartphone is not that important to the individual. At which scale should those intervals be computed?

Answering this is particularly complicated by the fact that at whatever scale we examine the data it appears with similar patterns (see Fig. 1a): short bursts are followed by long intervals. Quantitatively, the range of seconds to hours the distribution of inter-touch interval τ is scale free and can be described by a power-law distribution for intervals larger than $\tau_{\min}$ (see Fig. 1b)

$$p(\tau) \propto \tau^{-\alpha} \qquad \tau > \tau_{\min} \tag{1}$$

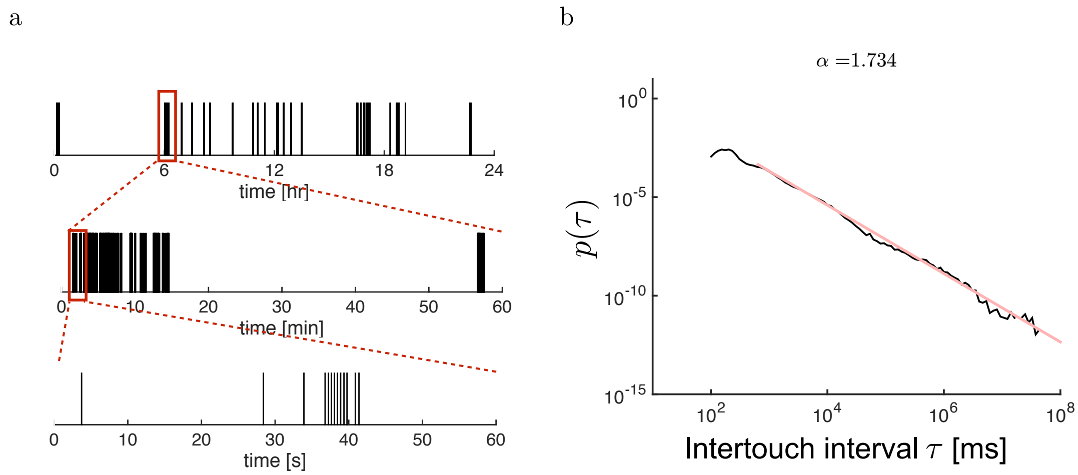

Figure 1: Intervals between smartphone touches are scale invariant. **a**. Smartphone touch activity during a given day (top) and hour (middle) and a minute (bottom). **b** The inter-touch interval distribution (black) is well captured by a power-law distribution (red). Data from one subject.

where α is the power-law exponent. This stands in contrast with point emission processes such as the Poisson process where zooming in or out would result in a distinct pattern across the different time-scales. Note that the range over which the distribution is consistent with a power-law is large and covers 4 decades i.e. from the onset of the power-law at $\tau_{\min}$ (which is on the order of 600ms) up the few hour range. The estimation of the power-law exponent from the sequence of intervals is done from a maximum likelihood approach (see Methods). Intuitively, a large α corresponds to the case where there are only few long intervals and therefore any non-smartphone activity will be interrupted by smartphone interactions. Therefore, we can expect that large α corresponds to high priority on the smartphone. This intuition is formalised in model proposed in the next section.

Model description

In this model, an individual can perform only two categories of tasks: either a task related to a smartphone screen TOUCH or OTHER task (see Fig. 2). Every task is associated with a priority level which is a number between 0 and 1. Let x denote the priority associated with the TOUCH task. x is drawn from the touchscreen priority distribution $p(x)$ which is given by

$$p(x) = (k+1)x^k \quad x \in [0,1], \quad -1 < k < \infty \tag{2}$$

where k is the *touchscreen priority index* [1]. If $k = 0$, $p(x)$ is a uniform distribution. If $k > 0$, then the distribution is pushed towards higher priorities x whereas if $k < 0$, the distribution has more mass for lower priorities. Note that if $k \in \mathbb{N}$, then $p(x)$ corresponds to the distribution of the maximal value of $k+1$ random numbers taken from a uniform distribution (between 0 and 1) which is the case considered by Oliveira and Vazquez (2009). We can therefore view our model as a generalisation[2] of the priority-based of Oliveira and Vazquez (2009) since we do not have the restriction that k is an

1. Getting a sample from this distribution is easy. It is sufficient to draw a random number r from a uniform distribution between 0 and 1 and set $x = r^{1/(k+1)}$.
2. From another angle, our model can also be seen as a simpler model than the model of Oliveira et al. since we consider a single agent instead of 2.

integer (because we do not restrict ourself to lists of tasks). The consequence of this generalisation is that our model can produce arbitrary power-law exponents whereas the Oliveira model produces only some rational exponents (e.g. their model can not produce a power-law exponent between 1.5 and 2).

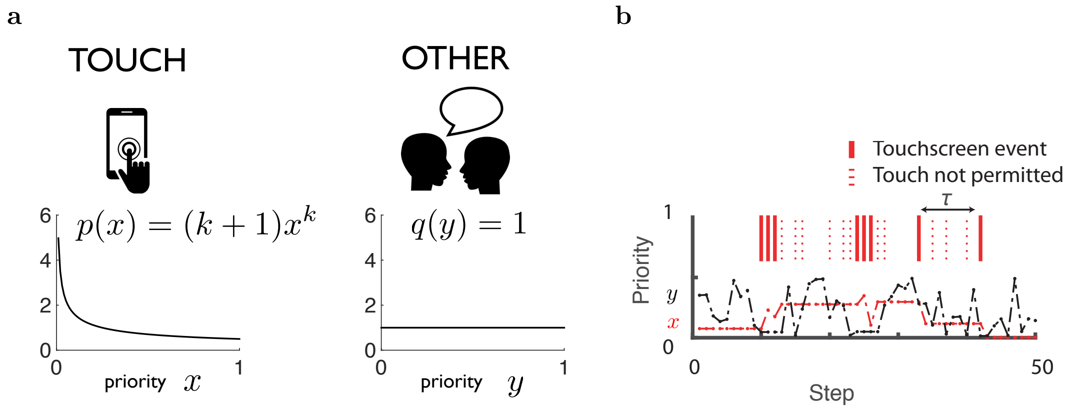

Figure 2: Model. **a** Priority distribution for the TOUCH tasks (left) and for the OTHER tasks (right). **b** Sample trace from the priority model.

Similarly, let y denote the priority associated with the OTHER task. y is drawn from the following distribution

$$q(y) = (k'+1)x^{k'} \quad y \in [0,1], \quad -1 < k' < \infty \tag{3}$$

Since we are interested in the relative priority of the TOUCH tasks compared to the OTHER tasks, we can set $k' = 0$ without loss of generality. We therefore have $q(y) = 1$.

Let E_t denote the presence ($E_t = 1$) or the absence ($E_t = 0$) of a TOUCH event at time t. The probability of generating such a TOUCH event at time t is a function of both the priority x_t of the TOUCH task and the priority y_t of the OTHER task.

$$p(E_t = 1 | x_t, y_t) = f(x_t, y_t) \tag{4}$$

For the sake of simplicity, we will assume that $f(x, y) = p$ if $x > y$ where p is called the *permission probability* and $f(x, y) = 0$ otherwise. Permission occurs with probability p and captures several aspects: the inability to use the smart phone because its battery is down or because the individual is in a meeting and cannot use his mobile phone. After the touch event, a new priority x is drawn from $p(x)$. If any of the two above conditions are not satisfied, then the OTHER task is chosen and therefore no TOUCH event is generated. In this case a new priority y is drawn from $q(y)$ and the procedure starts again. This detailed model is expressed in Table 1a. A sample trace of the evolution of x and y giving a sequence of touches is represented on Fig. 2c.

Note also that in the limit of $p \to 1$, the model becomes pathological in the sense that most of the inter-touch intervals are concentrated at the value of $\tau = 1$. The reason is the following. Let us assume that there is an event at time $t = 0$ where we have $x_0 > y_0$. If the next priority for the touchscreen task x_1 is larger than y_0, then there is a next event at $t = 1$ and the priority for the other task remains the same, i.e. $y_1 = y_0$. If, on the contrary $x_1 < y_0$, then at the time of the next event at time $t = \tau$, the priority for the other task y_τ will be smaller than the touchscreen task $x_\tau = x_1$ which is smaller than y_0. Therefore the value of y (at the time of an event) either stays the same or decreases. When it approaches zero, it becomes impossible for the priority x to fall below y and therefore there are events every time steps. By assuming that $p < 1$, the permission to touch the screen can be denied and therefore allow a potential increase in y at the time of the next event. The consequence of this $p < 1$ assumption is that the inter-event distribution is not anymore concentrated at $\tau = 1$.

The model predicts scale-invariant inter-touch interval distribution

By simulating this priority-based model, we can compute the inter-touch interval distribution and we find that the distribution is consistent with a power-law distribution (see Fig. 3a). The intuitive reason why this model produces heavy tails in the ITI distribution is that when a priority x is small, the random variable y needs to be drawn on average a very large number of times until it falls below x. This produces very long ITI. Actually, those long ITI are computationally expensive if we run simply the above detailed priority-based model. A much more efficient model (called the coarse-grain model – see Methods), takes advantage of the fact that we can analytically compute the probability of those long intervals given the priority x of the TOUCH task right after the last TOUCH.

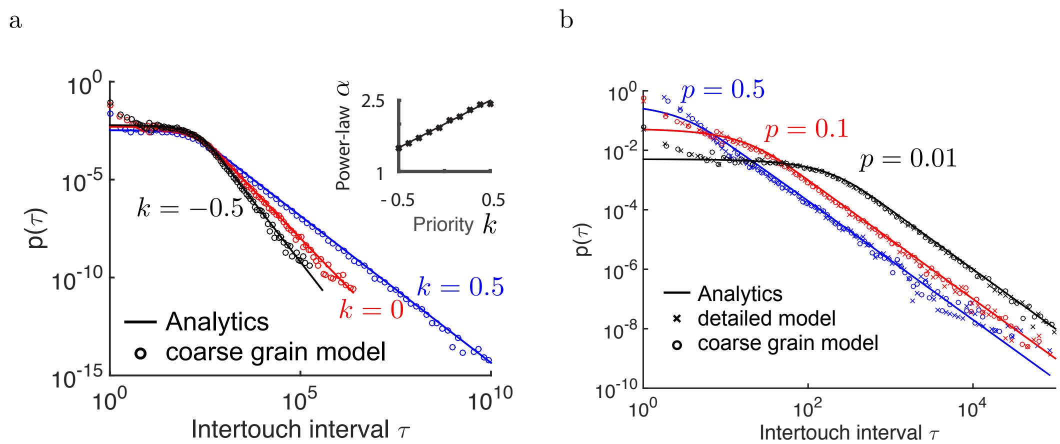

Figure 3: Properties of the model. **a** The model produces power-law inter-touch intervals. **Inset** The power-law exponent α is simply related to the priority index k with $\alpha = k + 2$. **b** The permission probability p does not influence the exponent, but shifts the start of the power-law distribution ($\tau_{\min}$). Note that the simulation results with the detailed model (crosses, see Table 1a) and with the coarse grain model (circles, see Table 1b) match well the analytical expression (line) from Eq. (13) for small p and large τ.

Smartphone priority can be estimated from ITI distribution

From the coarse grain model, we can calculate analytically the distribution of inter-touch interval $p(\tau)$ in the limit of small permission probability p and for large τ (see Methods):

$$p(\tau) \propto \tau^{-(k+2)} \tag{5}$$

So, through this result, it is possible to relate the power-law exponent α to the priority index k by the simple relation:

$$\alpha = k + 2 \tag{6}$$

The interest of this simple relationship is that for each individual, we can estimate the power-law exponent from the measured touch timings (see Methods) and therefore obtain the priority index k for this individual. By extracting the power-law exponent of each individual (see Fig. 4a), we found that the distribution of those power-law exponents is centred at $\alpha = 1.82$ with s.t.d $\sigma = 0.12$ (see Fig. 4b). We also found that 4.8% of the population has a power-law exponent larger than 2 - which corresponds to a priority index larger than 0, i,e, those individuals place a higher priority on their smartphone than on other tasks.

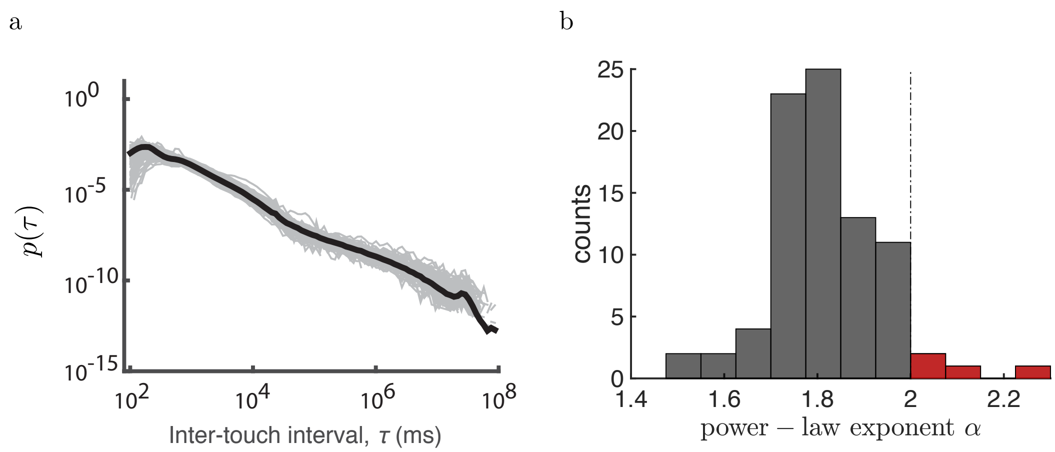

Figure 4: Inferring priority indices in a population of 84 subjects. **a** ITI distribution for each individual (gray lines) as well as the population ITI (black). **b** Number of subjects having a given power-law exponent α. 4.8% of the subjects have a power-law exponent larger than 2 (red), and therefore a priority index k larger than 0.

Conclusion

In this study, we proposed a model which allows to quantify the priority placed by individuals on their smartphone activity solely based on the touching times. In particular, this model directly relates the power-law exponent α of the ITI distribution to the priority index k by $\alpha = k + 2$. In particular, it sets an objective threshold $k^* = 0$ above which the smartphone tasks have higher priorities than other tasks.

Power-law distributions are ubiquitous in human dynamics. Indeed, the distribution of the interval between (a) two consecutive visits of of web portal by a single user, or between two emails sent out by a user or (c) two consecutive library leans made a by a single individual follow power-law distribution with exponent close to $\alpha = 1$ Vázquez et al. (2006). The distribution of the interval between the time a letter was received by Einstein, Darwin or Freud and the time they replied to that letter follows also a power-law distribution Vázquez et al. (2006); Oliveira and Barabási (2005) but with a power-law exponent of $\alpha = 3/2$. Curiously, the power-law exponent is $\alpha = 1$ for response time for emails Barabasi (2005). Power-law distributions are also ubiquitous in animal dynamics Reynolds (2011). The bouts of inactivity of the Drosophila fruit flies follow a power-law distribution with exponent of $\alpha = 2$ Martin (2004).

In all those cases, the power-law exponent is close to a rational number such as 1 or 3/2 and it is argued that the processes generating those intervals belong to some universality classes. In particular, the models are design in such a way that exponent is given by a rational number. For example, in the model proposed byOliveira and Vazquez (2009), the rational power-law exponent directly comes from the length [3] L of the task list $\alpha = 1 + 1/(L-1)$.

Our model is conceptually different in that it does not predict a rational power-law exponent. This is due to the fact that we assumed that the priority index can take any value (between -1 and ∞) and is not restricted to rational values. This is important for our model for three reasons. Firstly, we found that the distribution of α is not centred at a rational number such as 1, 3/2 or 2. Secondly, the amount of recorded touches is sufficiently large such that we can rule out the hypothesis that the spread of α is due to estimation uncertainty. Finally, by attributing to each individual a single α, we can highlight individual differences and determine to which extent those differences are related to other behavioral properties of individuals.

Methods

Subjects

A total of 84 individuals were recruited by using campus wide announcements at the University of Zurich and ETH Zurich. Ownership of a non-shared touchscreen smartphone with an android operating system was a pre-requisite for participation. All experimental procedures were approved according to the Swiss Human Research Act by the cantons of Zurich and Vaud. The procedures also conformed to the Helsinki Declaration. The volunteers provided written and informed consent to participate in the study.

Smartphone data collection

A custom-designed software application (app, Touchometer) that could record the touchscreen events with a maximum error of 5 ms Ghosh and Balerna (2016) was installed on each participant's phone. To determine this accuracy, controlled test touches were done at precisely 150, 300 and 600 ms while the Touchometer recorded at 147, 301 and 600 ms respectively, with standard deviations less than 15 ms (interquartile range less than 5 ms). The app posed as a service to gather the timestamps of touchscreen events that were generated when the screen was in an unlocked state. The operation was verified in a subset of phones by using visually monitored tactile events. The data was stored locally and transmitted by the user at the end of the study via secure email. One subject was eliminated as the app intermittently crashed after a software update. The smartphone data were processed by using MATLAB (MathWorks, USA)

Power-law exponent estimation

To extract the power-law exponent α and starting point of the power-law $\tau_{\min}$, we adopted the procedures developed previously Clauset et al. (2009). Briefly, for a given $\tau_{\min}$ the exponent α was estimated with a maximum likelihood estimator:

$$\hat{\alpha} = 1 + n \left(\sum_{i=1}^{n} \log \frac{\tau_i}{\tau_{\min}} \right) \tag{7}$$

3. Technically, Oliveira and Vazquez (2009) assume that an event occurs only if the interacting task for both agents A and B have the highest priority compared to all other tasks of length L_A (for agent A) and L_B for agent B. For simplicity I denoted $L = L_A = L_B$.

where τ_i, $i = 1, \ldots, n$ are observed intervals greater than $\tau_{\min}$. To estimate $\tau_{\min}$ the Kolmogorov-Smirnov (KS) statistic was used and the $\tau_{\min}$ with the least KS distance was chosen. The calculations were performed by using the MATLAB code provided by Aaron Clauset [4] but with minor modifications towards speedy processing of our data.

Corse-Grain Model

In order to prevent from drawing a prohibitively large number of times the random variable y, we propose a coarse grain model which dramatically speeds up simulation time. This coarse grain model follows the same approach than the one proposed in Oliveira and Vazquez (2009).

Let us assume that at time $t = 0$ an event has occurred, i.e. $x_0 > y_0$. At time step $t = 1$ a new priority $x_1 \equiv x$ is drawn from $p(x)$. If $x_1 > y_1$ (where $y_1 = y_0$) and the permission is granted, there is another event at $t = 1$. So the probability of having an interval of $\tau = 1$ depends only on y_0 whereas the probability of having an interval $\tau > 1$ will depend only on $x = x_1$. The idea of this coarse grain model is to calculate explicitly this probability distribution $Q(\tau|x)$ for $\tau > 1$ thereby avoiding to sample a large number of times the random variable y.

Let $\pi(x)$ denote the probability of an event at time $t > 1$ given that the priority for task T is x at time $t = 1$ (i.e. $x = x_1$). As mentioned above, an event can occur if two conditions are met, i.e. if $y < x$ and if the permission is given (with probability p). Since those 2 conditions are independent, we can write them as a product:

$$\pi(x) = p \int_0^x p(y)dy = px \tag{8}$$

where the last equality stems from the fact that we assumed that $k' = 1$. The distribution $Q(\tau|x)$ of ITI intervals for a given x is given by

$$Q(\tau|x) = \pi(x)(1 - \pi(x))^{\tau-2} \quad \tau > 1 \tag{9}$$

Note that this quantity is well normalized since $\sum_{\tau=2}^{\infty} Q(\tau|x) = 1$. This coarse-grain model is summarized in Table 1b. As we can see on Fig. 3b, the inter-touch interval distribution from the coarse grain model is in perfect agreement with the distribution obtained from the detailed model.

Calculation of the ITI

In the previous section, we saw that the probability of having an interval of $\tau = 1$ depends only on y_0 whereas the probability of having $\tau > 1$ depends on x_1. So, in order to calculate the overall inter-touch interval distribution $P(\tau)$, we need to average over the distribution $p(y_0|E_0 = 1)$ of priority values y_0 at the times of the events ($E_0 = 1$ denotes the fact that there is an event at time $t = 0$) for $\tau = 1$ and average over the distribution $p(x_1|E_0 = 1, E_1 = 0)$ of priority values x_1 given that there is an event at time $t = 0$ ($E_0 = 1$) and no event at time $t = 1$ ($E_1 = 0$) for $\tau > 1$:

$$p(\tau) = \begin{cases} p \int_0^1 p(x_1 > y_0) p(y_0|E_0 = 1) dy_0 & \text{if } \tau = 1 \\ \\ (1 - p(1)) \int_0^1 Q(\tau|x_1) p(x_1|E_0 = 1, E_1 = 0) dx_1 & \text{if } \tau > 1 \end{cases} \tag{10}$$

where the distribution $p(x_1|E_0 = 1, E_1 = 0)$ is given by

$$p(x_1|E_0 = 1, E_1 = 0) = \frac{p(E_1 = 0|x_1, E_0 = 1)}{\int_0^1 p(E_1 = 0|x, E_0 = 1) p(x) dx} p(x_1) \tag{11}$$

In the limit of small p, we have $p(E_1 = 0|x_1, E_0 = 1) \to 1$ and therefore

4. (http://tuvalu.santafe.edu/∼aaronc/powerlaws/)

a Detailed model

```
Input: p, k, N;
x ~ p(x); y ~ q(y);
n = 0, t = 0;
while n < N do
    t = t + 1;
    if x > y and Rand < p then
        n = n + 1; Event(n) = t;
        x ~ p(x);
    else
        y ~ q(y);
    end
end
Return: Event;
```

b Coarse Grain model

```
Input: p, k, N;
x ~ p(x); y ~ q(y);
n = 0, t = 0;
while n < N do
    n = n+1;
    if x > y and Rand < p then
        t = t + 1;
    else
        τ ~ Q(τ|x) % see Eq. 9 ;
        t = t + τ;
        y* ~ q(y), y = xy*;
    end
    Event(n) = t;
    x ~ p(x);
end
Return: Event;
```

Table 1: **a**. Detailed priority model which generates a list of N event times where the inter-event intervals are distributed according to a power-law with exponent $\alpha = k+2$. **b** Coarse grain model.

$$p(x_1|E_0 = 1, E_1 = 0) \simeq p(x_1) \tag{12}$$

For small p and for $\tau > 1$, we have

$$\begin{aligned} p(\tau) &\simeq \int_0^1 Q(\tau|x)p(x)dx \\ &= \int_0^1 px(1-px)(k+1)x^k dx \\ &= (k+1)p^{-(k+1)} \int_0^p x^{k+1}(1-x)^{\tau-2}dx \\ &= (k+1)p^{-(k+1)}\mathrm{Beta}\left(p; k+2, \tau-1\right) \end{aligned} \tag{13}$$

where $\mathrm{Beta}(z; a, b) = \int_0^z x^{a-1}(1-x)^{b-1}dx$ is the incomplete Beta function.

Note that when $\tau \gg 0$, the integrand of Eq. 13 is close to zero for $x > 1/(\tau-2)$ because of the term $(1-x)^{\tau-2}$. Therefore for $\tau \gg 1/p + 2$, we can extend the integration the interval from $[0, p]$ to $[0, 1]$:

$$p(\tau) \simeq (k+1)p^{-(k+1)}\mathrm{Beta}\left(k+2, \tau-1\right) \tag{14}$$

where $\mathrm{Beta}(a, b) = \int_0^z x^{a-1}(1-x)^{b-1}dx = \Gamma(a)\Gamma(b)/\Gamma(a+b)$ is the Beta function and $\Gamma(a) = \int_0^\infty t^{a-1}e^{-t}dt$ is the gamma function. If we set $\alpha = k+2$, we can write:

$$p(\tau) \simeq (\alpha-1)p^{-(\alpha-1)}\Gamma(\alpha)\frac{\Gamma(\tau-1)}{\Gamma(\alpha+\tau-1)} \qquad \tau \gg 1/p + 2 \tag{15}$$

Since $\Gamma(\tau+1) = \tau\Gamma(\tau)$, we have, for large τ, $\Gamma(\tau-1)/\Gamma(\alpha+\tau-1) \to \tau^{-\alpha}$. Therefore, the ITI distribut ion becomes, for large τ

$$p(\tau) \simeq (\alpha-1)p^{-(\alpha-1)}\Gamma(\alpha)\tau^{-\alpha} \tag{16}$$

which is a power-law distribution with exponent α. So the link between the priority index k which is a model parameter and the power-law exponent α which can be measured experimentally is given by

$$\alpha = k + 2 \tag{17}$$

and is independent of the permission parameter p (see also Fig. 3b). Here, again we stress the fact that since k can take any real value bigger than -1, α can take any real value bigger than 1 - which is in opposition to the Oliveira model where the power-law exponent can take only some specific rational values.

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